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Uncertainty Quantification of a Non-linear Rotating Plate Behavior in Compressible Fluid Medium

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Abstract

The paper focuses on application of uncertainty quantification techniques to a fluid structure interaction (FSI) problem involving a nonlinear rotating disc with surrounding fluid at high pressure without any dissipations. The deterministic dynamics shows strong influence of system parameters on the coupled FSI behavior. The nonlinear FSI problem has been discussed in detail and the instability mechanism has been presented. Such instabilities are known as flutter in closed medium and has significance in the turbomachinery industries. This gives way to a sudden large amplitude oscillation, known as the Hamiltonian Hopf bifurcation in such FSI systems. Uncertain variations in system parameters have been considered in order to see the effect on the flutter instability. Investigation with spectral uncertainty quantification tool has given the propagation of the input uncertainties on the instability behavior.

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Keywords: nonlinear rotating disc; compressible fluid; fluid-structure interaction; uncertainty quantification.

1. Introduction

Disc type structures in a closed fluid medium have wide ranging applications across many engineering disciplines. These include devices both on the macro and micro level. At the micro level, they have been used in a number of micro-electrochemical systems (MEMS), ultrasensitive force sensing devices, micro-gas turbines, etc. On the macro scale, they are used in data storage devices such as digital versatile discs, hard disc drives, compact discs, etc. Such systems can also effectively model the fluid structure interaction behavior in heavy engineering systems, such as turbines, centrifugal impellers.

A vibrating structure in the presence of the surrounding fluid medium will experience inertial, elastic and acoustic forces. The dynamic fluid-structure interaction (FSI) between nonlinear vibrations of a rotating disc structure and an acoustic system is the focus of our study here. This interaction arises either from the flow-induced vibrations of the structure or from the structure induced acoustic oscillations. The dynamics of the nonlinear rotating disc system alone is quite complex and in the presence of fluid, the resulting dynamical behavior can show significant deviation

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from the structural analysis in vacuum. The combined FSI system can be sensitive to a large number of system input parameters. Especially, in case of an impeller disc in centrifugal compressors, the surrounding high-density fluids significantly affect the disc dynamics and destabilize the disc through the flutter instability. Thus the design of the spinning disc for these kind of applications poses a significant challenge because of the interaction between the disc and the surrounding fluid.

Pretlove[1] was the first one to study the effect of the acoustic stiffness of a backed cavity on panel vibrations. Dowell *et al.* [2] had developed a comprehensive theoretical framework for these kinds of acousto-elastic interaction problems and this framework has been used as a ground work for the current study. Bokil *et al.* [3] developed a technique for the modal analysis of the acousto-elastic system based on Meirovitch's algorithm and applied this technique to a simple rectangular panel box system. Renshaw *et al.* [4] investigated the dynamics of the rotating disc in an enclosed compressible fluid both by experimental and theoretical means and concluded that the flutter instability of the rotating disc depends on the density ratio of fluid to disc and the geometry of the fluid enclosure. Kang and Raman [5] studied the flutter instability mechanisms of the flexible rotating disc immersed in a fluid enclosure in the context of the acoustic-structure problem and used Dowell's formulation for their work.

Magara *et al.* [6] investigated the shift in the natural frequencies of the centrifugal impeller disc due to the acoustic structure interaction between the fluid in the cavity and the impeller disc, both experimentally and numerically. By considering a small fluid enclosure, they have studied the effect of the fluid density on the coupled structure dominated frequencies. Magara *et al.* [7] further extended their study for rotating disc backed by an annular cavity.

The literature on nonlinear aero elastic instability mechanisms of a spinning disc is very limited, though geometric nonlinearity in such structures is a likely scenario. Hansen [8] and Jana *et al.* [9] studied both analytically and experimentally the nonlinear instability mechanism of a spinning disc in an unbounded enclosure by considering structural nonlinearity. While studying the instability mechanism analytically, Hansen [8] used the method of multiple scales which is more suited for weakly nonlinear cases, while Jana *et al.* [9] used the continuation and bifurcation software AUTO97 which is not restricted to weak nonlinear systems. But to the best of our knowledge, no such attempt was made to study the nonlinear interaction effects of the spinning disc in a bounded fluid environment. In this work, we attempt to study the nonlinear aero elastic instability mechanism of the spinning disc in the enclosed fluid using an analytical model. A deterministic model is developed first for the system including all the possible parameter which would affect its dynamics.

In the linear analysis of these FSI systems, changes in system parameters have shown significant effect on the combined system dynamics [5, 17], clearly there is a need to resolve the dynamics in the presence of nonlinearity as well. Moreover, engineering systems are likely to get influenced by the presence of parametric uncertainties. Quantifying the effect of such parametric uncertainties on the system response is important for the design and safety of such systems. Uncertainty Quantification is a field, which has been on a constant rise in the past two decades. Nonlinear FSI systems are known to be sensitive to physical uncertainties, since they often amplify the randomness of the parameter with time and also they can significantly affect the dynamical behavior such as changes the onset of the bifurcation points [10].

There are various techniques available in the literature for studying the propagation of uncertainty through a non-linear dynamical system. The Monte Carlo simulation (MCS) method gives good accuracy and convergence but at high computational cost as the number of required samples is very high. This has lead to development of many other methods which consume comparatively much lesser time than MCS and give results of comparable accuracy to that of MCS. One such method which has been applied in this paper is Polynomial Chaos Expansion (PCE). In this method, the stochastic quantity of interest is represented spectrally by employing orthogonal polynomials from the Askey scheme as a basis in the random space [11]. Using the standard Galerkin projection, this series expansion is incorporated into the stochastic differential equations and by orthogonal projection onto the various basis functions, we can elicit the differential equations without any random component [12]. In this Galerkin polynomial expansion, the governing equations are modified to a coupled form in terms of chaos coefficients. Hence the Galerkin polynomial chaos expansion is an intrusive approach. The modified governing equations are generally more complex and it is very tedious to derive them for some choice of uncertain parameters. In order to avoid this, several non-intrusive approaches have been proposed. Among these approaches, spectral method based PCE have been successfully applied in various FSI studies. In this method, a deterministic solver is repeatedly used at certain collocation points for a given

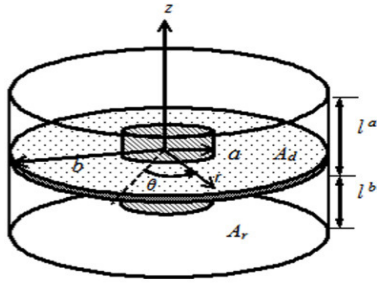


Fig. 1. A schematic diagram of the annular disc immersed in a compressible fluid filled cavity.

Table 1. Basic dimensions	of the system
Outer radius, b	4.74 cm
Inner radius, a	1.56 cm
Disc thickness, h	0.79 mm
Disc density, ρ_d	2700 kg/m ³
Fluid density, ρ_f	1.2 kg/m ³
Young's modulus, E	71 GPa
Poisson's ratio, ν	0.33
Speed of sound, c	343 m/s

distribution [13]. As compared to MCS, the number of collocation points are much lesser which makes this approach as a computationally cheaper one. The methodology is described later in this work.

2. Governing equation

The considered dynamic model comprises of a rotating annular disc placed in a cylindrical cavity filled with compressible fluid as in Fig. 1. The cavity is considered to be having rigid walls. The disc is of flexible material having thickness h and rotates about its axis at a constant angular velocity Ω . It is fixed/clamped at the inner radius a and free at the outer radius b . The fluid surrounding the disc is considered to be compressible and inviscid. All sources of dissipation are neglected in this study. As shown in Fig. 1, the upper and the lower cavity are separated by a flexible disc. The system parameters for the present analysis are inspired by the computational work of Kang *et al.* [5] and given in Table 1.

2.1. Disc equations

The field equations in the disc fixed coordinate system (r, Θ, t) for the rotating disc is derived based on the Von Karman plate theory for moderately large amplitude transverse vibrations and given by [14],

$$\rho h \frac{\partial^2 w}{\partial t^2} + D \nabla^4 w + \frac{1}{2} \rho h r^2 \Omega^2 \nabla^2 w + \rho h \Omega^2 r \frac{\partial w}{\partial r} = L[w, \phi], \quad (1)$$

$$\nabla^2 \phi - 2(1 - \nu) \rho h \Omega^2 = E h L[w, w], \quad (2)$$

where, the Bilinear operator L is defined as,

$$L[f, g] = \frac{\partial^2 f}{\partial r^2} \left[\frac{1}{r} \frac{\partial g}{\partial r} + \frac{1}{r^2} \frac{\partial^2 g}{\partial \Theta^2} \right] + \frac{\partial^2 g}{\partial r^2} \left[\frac{1}{r} \frac{\partial f}{\partial r} + \frac{1}{r^2} \frac{\partial^2 f}{\partial \Theta^2} \right] - 2 \left[\left(\frac{1}{r} \frac{\partial^2 g}{\partial r \partial \Theta} - \frac{1}{r^2} \frac{\partial^2 g}{\partial \Theta^2} \right) \left(\frac{1}{r} \frac{\partial^2 f}{\partial r \partial \Theta} - \frac{1}{r^2} \frac{\partial^2 f}{\partial \Theta^2} \right) \right], \quad (3)$$

$$\nabla^4 = \left(\frac{\partial^2}{\partial r^2} + \frac{1}{r} \frac{\partial}{\partial r} + \frac{1}{r^2} \frac{\partial^2}{\partial \Theta^2} \right)^2.$$

In equations (1) and (2), ϕ is the Airy stress function, w is the transverse displacement, ρ is the solid density, D is the flexural rigidity, ν is Poisson's ratio, h is disc thickness, Ω is disc rotation speed and E is modulus of elasticity. In Equation (1), on left hand side the first two terms are the same as the linear disc, the next two terms containing Ω are contributed due to rotation of the disc and on the right hand side, all other terms are contributed by the geometric non-linearity of the system. Equation (2) is the compatibility equation in polar coordinates. The equations (1) and (2) are converted to non-dimensional form and also to the ground fixed coordinate system (r, θ, t) using the following transformations (5),

$$r^* = \frac{r}{b}, \quad t^* = \frac{t}{t_0}, \quad w^* = \frac{bw}{h^2}, \quad t_0 = \sqrt{\frac{\rho h b^4}{D}}, \quad \kappa = \frac{a}{b}, \quad \Omega^* = \Omega t_0, \quad (4)$$

$$\theta = \Theta + \Omega t. \quad (5)$$

After transformation, they form the following equations (Asterisks have been dropped for convenience),

$$\frac{\partial^2 w}{\partial t^2} + \nabla^4 w + \frac{1}{2} \Omega^2 r^2 \nabla^2 w + \Omega^2 r \frac{\partial w}{\partial r} + 2\Omega \frac{\partial^2 w}{\partial t \partial \theta} + \Omega^2 \frac{\partial^2 w}{\partial \theta^2} = \epsilon L[w, \phi], \quad (6)$$

$$\nabla^2 \phi - \frac{2(1-\nu)}{\epsilon} \Omega^2 = L[w, w]. \quad (7)$$

Here, $\epsilon = \frac{12(-\nu^2)h^2}{b^2}$. The following are the boundary conditions for disc fixed at $r = \kappa$ and free at $r = 1$: the transverse displacement, slope, hoop displacement and radial displacement are zero at $r = \kappa$ (inner radius) and the bending moment, Kirchhoff edge reaction, radial stress and in-plane shear stress are zero at $r = 1$ (outer radius). Further as in equation (9), the Airy stress function ϕ has been written as a combination of two terms in which first term is due to disc rotation and the other one is due to nonlinearity as shown below in equation (8) [14],

$$\phi(r, \theta, t) = \frac{g(r)}{\epsilon} \Omega^2 + \phi_0(r, \theta, t), \quad (8) \quad w(r, \theta, t) \approx [\eta_n(t) \cos n\theta + \zeta_n(t) \sin n\theta] \psi_n(r). \quad (9)$$

By assuming single mode solution, the transverse displacement $w(r, \theta, t)$ can be written as in equation (9). Here, η_n and ζ_n are the generalized coordinates, n is the number of nodal diameter of the n^{th} mode and $\psi_n(r)$ is the radial part of the mode shape of the linear rotating disc. The eigenvalue problem of the linear rotating disc is a self-adjoint, the corresponding eigenfunctions are orthogonal. Hence, the disc radial mode shapes are normalized as,

$$\pi \int_{\kappa}^1 r \psi_{nm}(r) \psi_{kl}(r) dr = \delta_{nk} \delta_{ml}, \quad (10)$$

where, n and k are number of nodal diameters and m and l are the number of nodal circles. Using equations (7), (9) and the boundary conditions, $\phi(r, \theta, t)$ is found. Then, by substituting equations (8) and (9) in equation (6), multiplying the result with $r\psi_n(r) \cos n\theta$ and $r\psi_n(r) \sin n\theta$, and integrating the outcomes from $\theta = 0$ to $\theta = 2\pi$ and $r = 0$ to $r = 1$, we obtain the following equations,

$$\ddot{\eta}_n + 2n\Omega\dot{\zeta}_n + (\omega_n^2 - n^2\Omega^2)\eta_n = -\epsilon\alpha_n\eta_n(\eta_n^2 + \zeta_n^2), \quad (11) \quad \ddot{\zeta}_n - 2n\Omega\dot{\eta}_n + (\omega_n^2 - n^2\Omega^2)\zeta_n = -\epsilon\alpha_n\zeta_n(\zeta_n^2 + \eta_n^2), \quad (12)$$

where, ω_n is rotating disc frequency, α_n is cubic non-linear stiffness coefficient [14]. These equations (11, 12) are the discretized form of the governing equations of the nonlinear rotating disc.

2.2. Fluid equations

The propagation of infinitesimal disturbances in the inviscid and compressible fluid enclosed within the cavity satisfy the linear, homogeneous non-dimensionalised wave equation in the ground fixed coordinate system,

$$\nabla^2 \Phi - \frac{1}{C^2} \frac{\partial^2 \Phi}{\partial t^2} = 0, \quad C = \frac{c_0 t_0}{b}, \quad \Phi^* = \frac{\Phi t_0}{bh}, \quad (13)$$

where, ∇^2 is Laplacian operator, Φ is velocity potential, c_0 is the speed of sound and C is the ratio of speed of sound to disc flexural speed (b/t_0). Since the cavity walls are considered to be rigid, the normal component of velocity on the wall will be zero, $\frac{\partial \Phi}{\partial n} = 0$, where, n is the normal unit vector and A_r is the rigid wall surface area. In terms of the rigid wall acoustic normal modes $F_m(r, \theta, z)$ the fluid velocity potential Φ is written as in equation (14),

$$\Phi = \sum_m a_m(t) F_m(r, \theta, z), \quad (14) \quad \frac{1}{V} \int_V F_m F_n dV = M_{mn} \delta_{mn}, \quad (15)$$

where, $a_m(t) = \eta_{mnp} e^{i\Lambda_{mnp}t}$. Here, F_m , a_m and Λ_{mnp} are natural modes, generalized coordinates and natural frequencies of the acoustic cavity respectively. η_{mnp} is the superposition coefficient. The indices m, n and p denote the number of

nodal diameters, number of nodal circles and z directional node number of the mode shapes respectively. The rigid wall modes are orthogonal to each other and the orthogonality condition is given by equation (15). In equation (15), M_m is the acoustic modal mass of the m^{th} mode, V is the volume of the cavity and δ_{mn} is the kronecker delta function. By considering the orthogonality nature of the fluid modes, the field equation (13) is discretized using Green's theorem and Galerkin discretization technique and converted as,

$$M_m[\ddot{a}_m + \Lambda_m^2 a_m] = 0. \quad (16)$$

2.3. Coupled Problem

The uncoupled field equations (11), (12) and (16) are coupled by means of the interface boundary conditions. At the disc-fluid interface, the coupled system should satisfy the kinematic and static boundary condition as in equations (17) and (18) respectively,

$$\frac{\partial \Phi}{\partial n} = \mp \frac{\partial w}{\partial t} \quad \text{on } A_d, \quad (17) \quad Q = \Lambda \left[\frac{\partial \Phi^u}{\partial t} - \frac{\partial \Phi^l}{\partial t} \right] \quad \text{at } z = 0. \quad (18)$$

Here A_d is the disc surface area and it is flexible in nature, \mp sign indicates the opposing normal directions on the top and bottom side of the disc and $\Lambda = (\rho_f b / \rho_d h)$ is the mass ratio. In equation (18), Q represents the fluid pressure differential applied to the disc and the superscripts u and l denote upper and lower cavity velocity potentials. The disc is considered to be located at $z = 0$ plane. The coupled field equations for the m^{th} nodal diameter acoustic mode and the n^{th} nodal diameter structural mode are modified according to the kinematic boundary condition (17) and the forcing condition (18) based on technique adapted by Dowell *et al.* [2] as,

$$\ddot{\eta}_n + 2n\Omega\dot{\zeta}_n + (\omega_n^2 - n^2\Omega^2)\eta_n = -\epsilon\alpha_n\eta_n(\eta_n^2 + \zeta_n^2) + \Lambda A_d \left(\sum_m \dot{a}_m^c L_{nm}^c - \sum_m \dot{b}_m^c L_{nm}^c \right), \quad (19)$$

$$\ddot{\zeta}_n - 2n\Omega\dot{\eta}_n + (\omega_n^2 - n^2\Omega^2)\zeta_n = -\epsilon\alpha_n\zeta_n(\zeta_n^2 + \eta_n^2) + \Lambda A_d \left(\sum_m \dot{a}_m^s L_{nm}^s - \sum_m \dot{b}_m^s L_{nm}^s \right), \quad (20)$$

$$M_m^u[\ddot{a}_m^c + (\Lambda_m^u)^2 a_m^c] = -C^2 A_d [\dot{\eta}_n L_{nm}^c], \quad (21)$$

$$M_m^l[\ddot{b}_m^c + (\Lambda_m^l)^2 b_m^c] = C^2 A_d [\dot{\eta}_n L_{nm}^c], \quad (22)$$

$$M_m^u[\ddot{a}_m^s + (\Lambda_m^u)^2 a_m^s] = -C^2 A_d [\dot{\zeta}_n L_{nm}^s], \quad (23)$$

$$M_m^l[\ddot{b}_m^s + (\Lambda_m^l)^2 b_m^s] = C^2 A_d [\dot{\zeta}_n L_{nm}^s], \quad (24)$$

where, $L_{mn} = \frac{1}{A_d} \int_{A_d} F_m \psi_n dA$. In the above equations, s denotes the sine part, c denotes the cosine part, L_{mn} is the coupling coefficient, a_n and b_n are the generalized coordinates of the upper and lower cavities respectively. These equations were solved using a fourth order Runge-Kutta method. The major difference between coupled and uncoupled equations is the addition of an extra term L_{mn} which is the coupling coefficient between the m^{th} acoustic mode and the n^{th} structural mode. The equation leads us to a rule that an acoustic and structural mode will couple only if nodal diameters of the corresponding uncoupled modes are alike. This helps in making the computations inexpensive, thus, giving us a better insight of the problem.

3. Nonlinear FSI problem

From the linear FSI analysis in the literature, it has been found that the system shows significant sensitivity to rotation speed, fluid density and speed of sound of the fluid medium [5, 6, 17]. In the study that follows, the effect of the disc rotation speed on the nonlinear FSI problem is studied deterministically and the effect of uncertainties associated with the fluid density and the speed of sound on the system response are studied using a technique called uncertainty quantification (UQ) technique. The FSI behavior of a nonlinear disc in a bounded fluid medium has not been studied in the literature, which forms the deterministic part of the investigation in this paper.

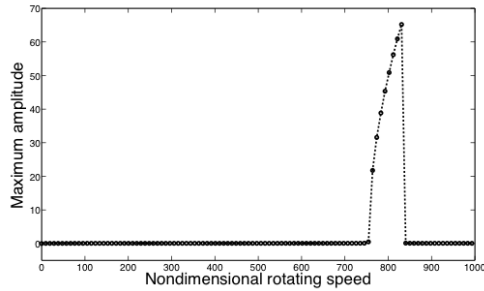


Fig. 2. Instability diagram - maximum amplitude of generalized displacement (ζ) v/s non-dimensional rotating speed.

The equations (19-24) are solved for various disc rotation speeds ranging from 0 to 1000. The maximum amplitude of the two nodal diameter mode of the backward travelling wave [5] with the rotation speed is given in Fig. 2. The figure shows that for lower rotating speeds ($\Omega < 754$), the maximum amplitude remains the same and is equal to the initial condition ($\zeta_0 = 0.05$). The system being undamped, no growth in the amplitude indicates a stable region up to $\Omega \approx 754$. As the rotation speed is increased further, we see a sudden growth in the amplitude, which signifies instability in the system. Since the undamped gyroscopic systems are Hamiltonian, the rapid increase in the amplitude indicates a Hamiltonian Hopf bifurcation with the rotation speed as the bifurcation parameter [15]. The bifurcation point is referred as the disc flutter onset speed. However, we see

that for rotation speeds greater than 840, the maximum amplitude is again same as that of the initial condition. This indicates a transition back to the stable region. The flutter onset and the instability range are same as in linear analysis [17]. This bifurcation behavior is studied in the next section in the presence of parametric uncertainties.

4. Polynomial chaos expansion(PCE)

Polynomial chaos expansion is a spectral uncertainty quantification (UQ) tool in which the stochastic quantities are projected using a family of orthogonal polynomials in the random space. The polynomials are chosen from the generalized Askey scheme. A projection based non-intrusive version of PCE has been used here. In the PCE form, a stochastic quantity of interest can be written as the following,

$$X(t, \xi) = \sum_{j=0}^k \hat{X}_j(t) \tau_j(\xi(l)), \quad (25) \quad \hat{X}_j(t) = \frac{\langle X(t, l), \tau_j \rangle}{\langle \tau_j, \tau_j \rangle}, \quad j = 0, 1, \dots, n, \quad (26)$$

where, τ_j is the polynomial of j^{th} order, ξ is the random variable and \hat{X}_j are the deterministic coefficients, which can be determined by equation (26). The denominator is known as the polynomials which are orthogonal. The weighing function depends on the orthogonal polynomials being used; for Hermite polynomials the associated function is the Gaussian distribution function and the orthogonality condition becomes, $\langle \tau_j, \tau_j \rangle = j!$ and $\langle a, b \rangle$ represents the standard inner product. The inner product in the numerator is evaluated using a tensor product quadrature as follows [16],

$$\langle X(t, \xi_k), \tau_j \rangle = \sum_{k=0}^p X(t, \xi_k) \tau_j(\xi_k) W_k, \quad \text{where } p \geq n \quad (27)$$

where, p is the total number of collocation points, ξ_k is the k^{th} collocation point and W_k is the weight function corresponding to k^{th} collocation point. The results from PCE will become more accurate as the order of expansion increases but it will also increase the computational cost so we need to find the minimum order of expansion at which PCE results converge. This convergence is validated with standard MCS. In the present case, C (Ratio of Speed of sound) and Λ (Weighted Mass Ratio) have been taken as two independent random variables with Gaussian distribution. Here we will solve only for one random variable at a time. Since the random variable has Gaussian distribution, Hermite polynomials will be used. The mean μ and standard deviation σ values are mentioned below,

$$C = \mu_c + \sigma_c \xi = 343.9332 + 0.5\xi, \quad \Lambda = \mu_\Lambda + \sigma_\Lambda \xi = 0.045 + 0.013\xi. \quad (28)$$

5. Results and discussions

The effect of the randomness associated with the input parameters on the system response is quantified in terms of response statistics like probability density function (PDF). For the Gaussian input C , we plotted probability density

function (PDF) for the generalized displacement ζ at time $t = 15$ s and $t = 30$ s using both PCE (of different orders) and MCS which gave us an insight into the accuracy and efficiency of PCE. The same computations were performed for weighted mass ratio Λ as a Gaussian input. As shown in Fig. 3, at $t = 15$ s, PCE of order 5 and 10 show good match with standard MCS. Whereas at $t = 30$ s, a reasonable agreement with MCS is obtained only with PCE of order 15. This means that with time, the same order of chaos polynomials are not able to capture the response, and higher orders are needed. This problem is known as degeneracy. This degeneracy is found in classical wing flutter as well [10]. The same can be observed in case of Λ as a random variable in Fig. 4 in which at $t = 15$ s, convergence occurs at 10th order whereas at $t = 30$ s, convergence occurs at 15th order. Here, a bimodal behavior in the PDF is seen to be emerging. This is due to the phase shifting behavior between the realizations [16]. This bimodal behavior should be more prominent with time. Time consumed for solving MCS is 16.1 days of real time, which was very high as compared to PCE, which took 19 hours or 0.792 days for a 40th order. The stochastic bifurcation plot as a function

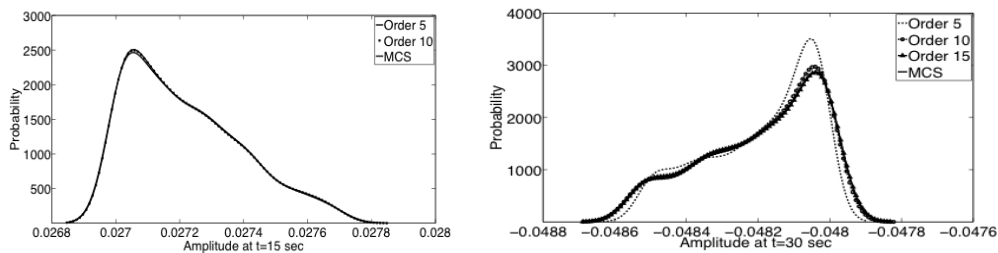


Fig. 3. PCE and MCS convergence for C at $t = 15$ secs and $t = 30$ secs.

of a bifurcation parameter Ω for random C and Λ are shown in Fig. 5(a) and Fig. 5(b). In both the figures, three bifurcation behaviors are plotted. The minimum and maximum branches corresponds to the minimum and maximum values of the random variable (RV) in the distribution range. In both the cases, the onset of the instability and the instability range are shifted due to the randomness in the input parameter.

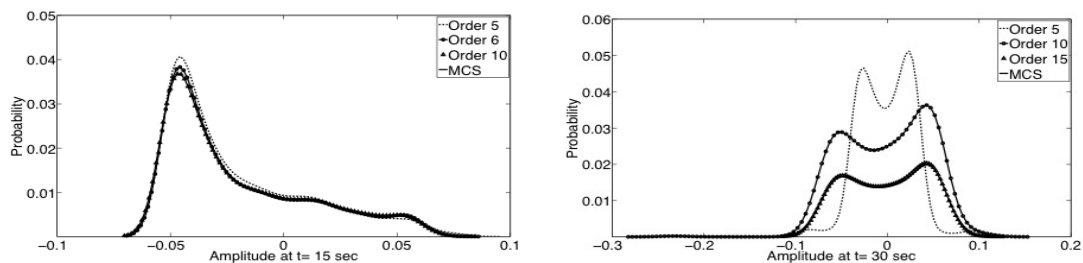


Fig. 4. PCE and MCS convergence for Λ at $t = 15$ secs and $t = 30$ secs.

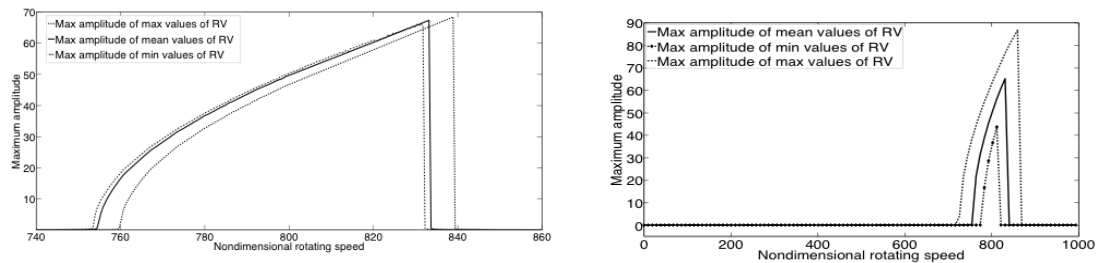


Fig. 5. (a) Random C : stochastic bifurcation plot; (b) Random Λ : stochastic bifurcation plot.

6. Conclusion

The classical acousto-elastic formulation for a rotating disc surrounded by a bounded fluid medium has been extended to a system that account for the geometric nonlinearity of the disc. The nonlinear effects arise due to moderately large amplitude oscillations of the disc. By keeping disc rotation speed as a bifurcation parameter, the nonlinear effects on the disc oscillations are studied in detail. The study shows the existence of an instability region beyond a certain speed where the specific mode restabilizes. The instability in a Hamiltonian system represents a Hamiltonian Hopf bifurcation. The effect of uncertainties associated with the input parameters on the system response is studied using a spectral method based Polynomial Chaos Expansion (PCE) and the results are validated against standard Monte Carlo simulations (MCS). It is also observed that the computational time of PCE is significantly lesser compared to MCS without any loss of accuracy. It is evident from the study that the randomness associated with the speed of sound and the fluid density shift the flutter onset and the instability range. By considering multi mode approximation for disc oscillations and including fluid nonlinearities, the study will be extended further in future.

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